An Optimal Solution to an Assignment Problem Using Zero Neighbouring Method

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Abstract

Assignment problem is a type of optimization problem which deals with allocation of different resources to different activities on one-to-one basis. It has a lot of relevance in practical life as many real world problems are formulated in the form of an assignment problem. In this research paper, a different approach (other than the existing methods in literature) for solving an assignment problem has been introduced along with the algorithm and numerical example.

Key Words: Assignment problem, Zero neighbouring method, Hungarian Algorithm, Optimization.

1. Introduction

A major challenge faced today by business organizations is that of allocation of resources that can be put to different uses. Every individual does not have the same ability to perform a given job. Different persons have different abilities to execute the same task and these different capabilities are expressed in terms of cost/profit/time involved in executing a given job. Therefore, the need arises to work on the constraints of assigning different workers to different jobs so that the cost of performing such jobs is minimized thereby providing more outcome with the resources available than just reacting to the demands of the moment.

The assignment problem is a type of optimization problem that deals with the assignment of different tasks to different workers with an objective to minimize the cost of assigning the task. It is a special type of transportation problem wherein the supply at each origin is one unit and the demand at each destination is one unit.

Various methods have been developed to find an optimal solution to the assignment problem. Dantzig [1] formulated the assignment problem as a linear programming problem and solved it using simplex method. Kuhn [2] proposed a method , popularly known as the Hungarian method, of solving an assignment problem. Later on, numerous methods were proposed for solving an assignment problem (see [4]-[9]).

Thiagarajan, K., et al. [5] proposed a Zero Neighbouring method to find an optimal solution to a Transportation problem. In this paper, we have used Zero Neighbouring method to find an optimal solution to an assignment problem. This method is easy to understand and use. Also it helps in finding an optimal solution to an assignment problem in limited number of iterations.

The rest of the paper is organized as follows. Section 2 presents the mathematical formulation of Assignment problem. Section 3 shows the steps of Zero Neighbouring method. In section 4, a numerical example is solved using the steps of zero-neighbouring algorithm and the results are compared with the Hungarian Method. The conclusion is presented in Section 5.

2. Mathematical Formulation of Assignment Problem

Let there be n tasks that are to be performed by n workers so that one job is assigned to only one worker and define

$$x_{ij} = \begin{cases} 1, & \text{if worker } i \text{ is assigned job } j \\ 0, & \text{otherwise} \end{cases}$$

Then the mathematical formulation of the assignment problem is

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad , i = 1,2,..........,n.$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad , j = 1,2,..........,n.$$

$$x_{ij} = 0 \text{ or } 1$$

where *cij* is the cost of assigning worker *i* to job *j*.

3. Algorithm of Zero Neighbouring Method

Consider a balanced assignment problem with n rows and n columns. The steps of Zero Neighbouring method are as follows:

1. Select the minimum element from each row and subtract it from all the elements of the corresponding row.
2. Select the minimum element from each column and subtract it from all the elements of the corresponding column.
3. Now each row and each column has at least one zero. In the reduced matrix, corresponding to each zero, find *sij* where

   sij = average of all the costs in the cells adjacent to zeros
4. Select the cell with highest value of *sij*. If there is a tie in the value of *sij*, break the ties arbitrarily. Assign i^{th} person to j^{th} job.
5. Delete i^{th} row and j^{th} column. The resultant matrix must possess at least one zero in each row as well as in each column, else repeat steps 1 and 2.
6. Repeat steps 3 to 5 until all the persons are assigned a job.

4. Numerical Example

Illustration 1. Consider the following problem of assigning four tasks W,X,Y,Z to four workers A,B,C,D. The cost (in thousand rupees) of performing each job by each worker is given in the table below.

	W	X	Y	Z
A	7	9	8	13
B	16	16	15	11
C	16	19	10	15
D	16	17	14	6

4.1. Optimal solution by Zero-Neighbouring method

After using step 1 and 2 of the algorithm on Illustration 1, the following matrix/table is obtained

	W	X	Y	Z
A	0	0	2	5
B	5	3	5	0
C	5	6	0	4
D	2	1	0	2

Applying Step 3 and calculating the suffix, *sij*, corresponding to each zero, the resultant matrix/table is

	W	X	Y	Z
A	$0_{2.5}$	$0_{1.3}$	1	6
B	5	3	4	0_{5}
C	6	7	0_{4}	5
D	2	1	0_{1}	2

From all the *sij's* obtained , the maximum value of *sij* is corresponding to the cell (B,Z). Therefore, worker B is assigned job Z. Eliminating the row corresponding to B and column corresponding to Z, the resulting matrix/table is

	W	X	Y
A	0	0	1
C	6	7	0
D	2	1	0

As each row and each column has at least one zero, finding *sij's* corresponding to zeros, we have

	W	X	Y
A	0_{3}	$0_{2.7}$	1
C	6	7	$0_{2.7}$
D	2	1	$0_{0.5}$

From among the *sij's* obtained, the maximum *sij* is corresponding to the cell (A,W). Therefore, worker A is assigned job W. Eliminating the row corresponding to A and column corresponding to W, the resulting matrix/table is

	X	Y
C	7	0
D	1	0

As the first column does not have a zero, using step 2 of the algorithm we get the following matrix/table,

	X	Y
C	6	0
D	0	0

Using Step 3 of the algorithm the *sij's* corresponding to zeros are

	X	Y
C	6	0_{3}
D	0_{3}	0_{0}

As there is a tie between the maximum value of *sij's*, so using Step 4 of the algorithm we arbitrarily select cell (C,Y) i.e. worker C is assigned Job Y. As after eliminating the row corresponding to C and column corresponding to Y, we are left with only one cell i.e. (D,X), therefore, worker D is assigned job X.

Thus, the Zero Neighbouring method gives the following assignment :

Worker	Job	Cost (in thousand rupees)
A	W	7
B	Z	11
C	Y	10
D	X	17
Total Cost = Rs. 45000		

4.2. Optimal Solution by Hungarian Algorithm

After using row and column reduction on Illustration 1, the matrix/table obtained is

	W	X	Y	Z
A	0	0	2	5
B	5	3	5	0
C	5	6	0	4
D	2	1	0	2

Making the initial assignment, we have

	W	X	Y	Z
A	[0]	~~0~~	2	5
B	5	3	5	[0]
C	5	6	[0]	4
D	2	1	~~0~~	2

As worker D has not been assigned any job, so the solution is not optimal. Drawing the minimum number of lines to cover all zeros, we have

	W	X	Y	Z
A	[0]	~~0~~	2	5
B	5	3	5	[0]
C	5	6	[0]	4
D	2	1	~~0~~	2

As the number of lines are less than the order of the matrix so we will select minimum element from the uncovered elements and subtract it from all the uncovered elements and add it to the elements that are at the intersection of two line. Thus, the revised matrix/table is

	W	X	Y	Z
A	0	0	3	5
B	5	3	6	0
C	4	5	0	3
D	1	0	0	1

Doing the assignment in the above matrix/table, we have

	W	X	Y	Z
A	[0]	~~0~~	3	5
B	5	3	6	[0]
C	4	5	[0]	3
D	1	[0]	~~0~~	1

As the number of assignments are equal to the order of the matrix and in each row as well as each column there is one and only one assignment so an optimal solution to the given problem has been attained. The optimal assignment along with the optimal cost is:

Worker	Job	Cost (in thousand rupees)
A	W	7
B	Z	11
C	Y	10
D	X	17
Total Cost = Rs. 45000		

5. Conclusion

The application of Zero Neighbouring method to solve an assignment problem is discussed in this paper. This approach is simple to understand and easy to use. In certain cases, it has been observed that zero-neighbouring method obtains an optimal solution in a fewer number of steps as compared to Hungarian Algorithm. A numerical example is solved using Zero-Neighbouring method to show its efficacy. Also the optimal solution obtained by this method is compared with the optimal solution obtained by Hungarian Algorithm to show its validity.

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